Mathematics Education: Issues and Trends Across the States.

Achieve

Achieve is an independent, nonpartisan, nonprofit education reform organization dedicated to working with states to raise academic standards and graduation requirements, improve assessments, and strengthen accountability.

Achieve

- We provide technical assistance
- We convene states, experts, and partners
- We conduct research
- We develop advocacy resources

- When does *proficiency* mean in your classes?

- Do your grades reflect that?

Teaching and Learning Mathematics

- Ways of doing
- Ways of thinking
- Habits of thinking

Ways of Doing?

$$2014 - 1941$$

Step 1: Notice the subtraction exercise.

$$2014 - 1941$$

Step 2: Set up the problem in a template.

$$\begin{array}{r} 2014 \\ \underline{-1941} \\ \\ \end{array}$$

Step 3: Work from right to left. Subtract 1 from 4.

$$\begin{array}{r} 2014 \\ \underline{-1941} \\ 3 \end{array}$$

Step 4. Got it. Now move one column to the left. Subtract 4 from 1.

Step 5. Realize that I cannot subtract 4 from 1.

Step 6. Remember that I need to "borrow" from the next column to the left.

Step 7. Notice there is nothing to borrow. (Perhaps also consider how I wasn't going to ever give it back anyway.)

Step 8. Borrow from the 2 in the far left column. Make it a 1.

Step 9. Think of the "0" in the top row as a 10. Wedge a tiny 1 between the 2 and the 0. Some might want to make it a little higher than the 0.

Step 10. Now borrow from the newly made 10. Make it a 9 by crossing out the 0 and 1 you just created during the previous step.

Step 11. Wedge a tiny 1 to the left of the original 1. I now have something that looks like this:

$$\begin{array}{r} 1\ 9 \\ \cancel{2}\cancel{0}1\ 1\ 4 \\ -\ 1\ 9\ 4\ 1 \\ \hline 3 \end{array}$$

Step 12. Subtract 4 from the newly-made 11. Get 7.

Step 13. Subtract 9 from 9.

Step 14. Subtract 1 from 1.

Step 15. Declare the answer to be 73.

SCHOOL INSTRUCTOR

~~Chinese TV~~ employs robot as ~~weather reporter~~; ~~anchors~~ worried

By PTI | 24 Dec, 2015, 12.43PM IST Post a Comment

TEACHERS

http://economictimes.indiatimes.com/news/international/world-news/chinese-tv-employs-robot-as-weather-reporter-anchors-worried/articleshow/50308175.cms 1/4/16

Jordan Ellenberg:

But, too often, we teach our students that “doing mathematics” means “manipulating clusters of digits according to rules presented to us by the teacher.”

That’s not math. And when we teach our students to do that, and only that, we are training them to be slow, buggy versions of Excel. What’s the point?

http://www.slate.com/blogs/how_not_to_be_wrong/2014/06/03/number_sentences_stephen_colbert_thinks_they_re_silly_they_re_not.html

Paul Lockhart

"The main problem with school mathematics is that there are no problems. Oh, I know what passes for problems in math classes, these insipid "exercises." "Here is a type of problem. Here is how to solve it. Yes it will be on the test. Do exercises 1-35 odd for homework." What a sad way to learn mathematics: to be a trained chimpanzee."

-Paul Lockhart

https://www.maa.org/external_archive/devlin/LockhartsLament.pdf (According to Devlin this was originally written in 2002.)

Ways of Thinking?

Question:

How many years was it from December 7, 1941 to December 7, 2014?

How did you figure it out?

One possibility:

- There are 59 years from 1941 to 2000.
- There are 14 years from 2000 to 2014.
- That means there are 59+14 = 73 years between 1941 and 2014.

Another:

Shift the whole thing by one year. The elapsed time from 1941 to 2014 is the same as 1940 to 2013. From here a counting up method is *exceptionally* efficient.

But what does it take to think like this?

2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Source: Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

3.NBT

2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Source: Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

4.NBT

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Source: Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Calling a truce in the math wars

"That afternoon, Fortmann put concepts in my head that I'd never really thought about. For instance, that subtraction is just "un-addition," the ultimate distance between two numbers. In my head, negative numbers turned from a series of tricks to memorize into a concept I could visualize."

-Joanna Weiss

https://www.bostonglobe.com/opinion/2016/01/08/calling-truce-math-wars/ATydAt9bjxVrKyvR88WnBP/story.html#comments retrieved 1/8/2016

Paul Lockhart

"The truly painful thing about the way mathematics is taught in school is not what is missing— the fact that there is no actual mathematics being done in our mathematics classes— but what is there in its place: the confused heap of destructive disinformation known as "the mathematics curriculum."

-Paul Lockhart

https://www.maa.org/external_archive/devlin/LockhartsLament.pdf (According to Devlin this was originally written in 2002.)

2015 CUPM Curriculum Guide

Mimicking procedures is not enough. Students must experience for themselves what it means to understand mathematical ideas deeply. They must feel the exhilaration of mathematical insight. They must learn and appreciate productive struggle. They must learn to articulate mathematical ideas clearly, orally and in writing.

http://www.maa.org/sites/default/files/pdf/CUPM/pdf/CUPMguide_print.pdf (pp 62)

2015 CUPM Curriculum Guide

Like virtually all college faculty, they teach the way they were taught. But, ironically (and embarrassingly),it would be difficult to design an educational model that is more at odds with the findings of current research about human cognition than the one being used today at most colleges and universities

http://www.maa.org/sites/default/files/pdf/CUPM/pdf/CUPMguide_print.pdf (p. 70 quoting Halpern and Hakel (2003))

K-12 State Standards

- Achieve considers six criteria:
  - Focus
  - Coherence
  - Rigor
  - Clarity
  - Specificity
  - Measurability

K-12 State Standards

- Achieve considers six criteria:
  - Focus
  - Coherence
  - Rigor
    - Procedural skill and fluency
    - Conceptual understanding
    - Application
  - Clarity
  - Specificity
  - Measurability

Mile Wide, Inch Deep (2002)

"Our intended content is not focused…If you look at U.S. textbooks, you'll find there is no textbook in the world that has as many topics as our mathematics textbooks, bar none."

Schmidt, W., Houang, R., & Cogan, L. (2002). A Coherent Curriculum: The case of mathematics. American Educator 26(2), 10-26. https://www.aft.org/sites/default/files/periodicals/curriculum.pdf. P.3.

Mile Wide, Inch Deep (2002)

"Our intended content is highly repetitive. We introduce topics early and then repeat them year after year. To make matters worse, very little depth is added each time the topic is addressed because each year we devote much of the time to reviewing the topic."

Schmidt, W., Houang, R., & Cogan, L. (2002). A Coherent Curriculum: The case of mathematics. American Educator 26(2), 10-26. https://www.aft.org/sites/default/files/periodicals/curriculum.pdf. P.3.

Mile Wide, Inch Deep (2002)

"Our intended content is incoherent. Math, for example, is really a handful of basic ideas; but in the United States, mathematics standards are long laundry lists of seemingly unrelated, separate topics."

Schmidt, W., Houang, R., & Cogan, L. (2002). A Coherent Curriculum: The case of mathematics. American Educator 26(2), 10-26. https://www.aft.org/sites/default/files/periodicals/curriculum.pdf. P.3.

“The [Common Core] standards were not so much assembled out of topics as woven out of progressions. “

The Structure is the Standards. Phil Daro, William McCallum, and Jason Zimba, February 16, 2012.
http://commoncoretools.me/2012/02/16/the-structure-is-the-standards/

achievethecore.org/coherence-map/#

COHERENCE MAP

Kindergarten

1st Grade

2nd Grade

3rd Grade

4th Grade

5th Grade

6th Grade

7th Grade

8th Grade

National Mathematics Advisory Panel (2008)

"To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem solving skills. Debates regarding the relative importance of these aspects of mathematical knowledge are misguided."

National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education. https://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf.

- Focused
- Coherent
- Rigorous

Rigor

- Procedural skill and fluency
- Conceptual understanding
- Application

Aspects of Rigor Matrix (abbreviated)	The item does not involve application.	The item involves an application
The item targets procedural skill expected by the grade level.	PSF	PSF-APP
The item targets conceptual understanding[4] and procedural skill expected by the grade level OR targets conceptual understanding but can also be answered using at least some procedural skill expected by the grade level.	C-PSF	C-PSF-APP
The item targets conceptual understanding. Students may explain, strategize, evaluate, determine, compare, or classify.	C	C-APP

Teaching and Learning Mathematics

- Ways of doing
- Ways of thinking
- Habits of thinking

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the understanding of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

- These are essential aspects of being proficient.

State Standards

- Many states have engaged in a standards review process.
- Most states that begin with the Common Core end up with compatible standards.
- Teachers can still share
  - A common language
  - Materials

State Standards - Observations

- Some high school standards “for all” moved to fourth year courses.
- Postponement of high school statistics topics.

Even with similar standards…

- High quality materials remain difficult to find.

Rand (2016)

Figure 2.7
Top Online Resources Consulted for Instruction Among Mathematics and ELA Teachers in SACC States

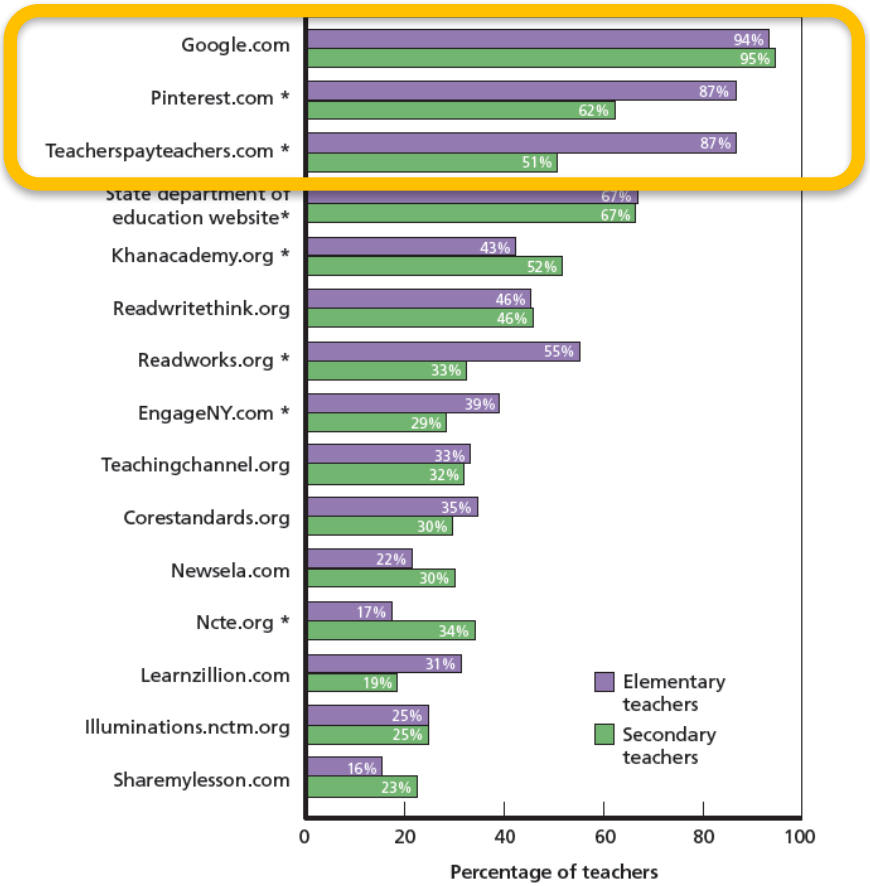
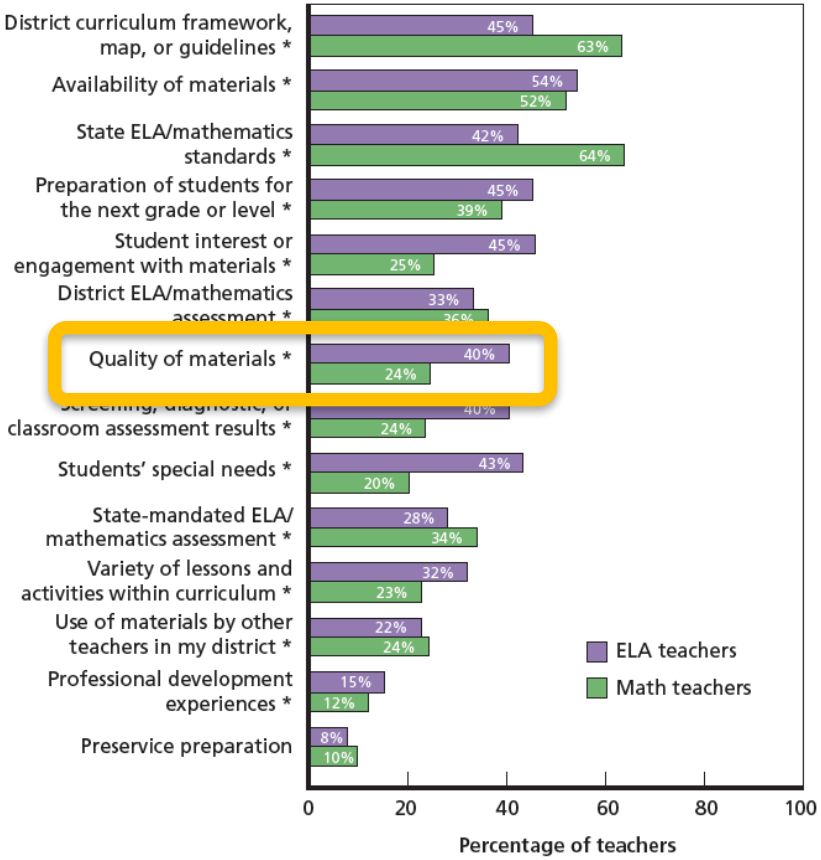


Figure 2.11
Factors Influencing Mathematics and ELA Teachers' Use of Instructional Materials "A Great Deal" in SACC States



Opfer, V. D., Kaufman, J. H., & Thompson, L. E. (2016). Implementation of K–12 State Standards for Mathematics and English Language Arts and Literacy. *RAND Corporation*. Pp. 39, 45. Available at http://www.rand.org/content/dam/rand/pubs/research_reports/RR1500/RR1529/RAND_RR1529.pdf

Quality materials matter

"In terms of achievement impacts, our findings suggest non-trivial gains in student achievement are attainable simply by choosing more effective curriculum materials."

Corey Koedel and Morgan Polikoff

Koedel, C., & Polikoff, M. (2017) Big bang for just a few bucks: The impact of math textbooks in California. *Brookings*. p. 1. Available at https://www.brookings.edu/wp-content/uploads/2017/01/es_20170105_polikoff_evidence_speaks1.pdf

Higher Education

The National Research Council (2013)

"The need to create a truly compelling menu of creatively taught lower-division courses in the mathematical sciences tailored to the needs of twenty-first century students is pressing…"

"Most mathematics departments still tend to use calculus as the gateway to higher-level coursework, and that is not appropriate for many students."

National Research Council. (2013). The mathematical sciences in 2025. *National Academies Press*. Pp. 10, Available at https://www.nap.edu/catalog/15269/the-mathematical-sciences-in-2025.

The INGenIOuS Project (MAA)

- "Alternative curricular entry points (e.g., courses other than freshman-level algebra or beginning calculus) and pathways to undergraduate and graduate degrees should be developed."

Zorn, P., Bailer, J., Braddy, L., Carpenter, J., Jaco, W., & Turner, P. (2014). The INGenIOuS project: Mathematics, statistics, and preparing the 21st century workforce. *The Mathematical Association of America*. p. 20. http://www.maa.org/sites/default/files/pdf/ingenious/INGenIOuS-report.pdf

A Common Vision (MAA)

"The current attention to big data and the demand for college graduates with data skills should prompt changes in our entry-level courses which result in students being better prepared for jobs requiring computational and statistical skills. Thus, there is a call to provide mathematically substantive options for students who are not headed to calculus. These entry courses should focus on problem solving, modeling, statistics, and applications."

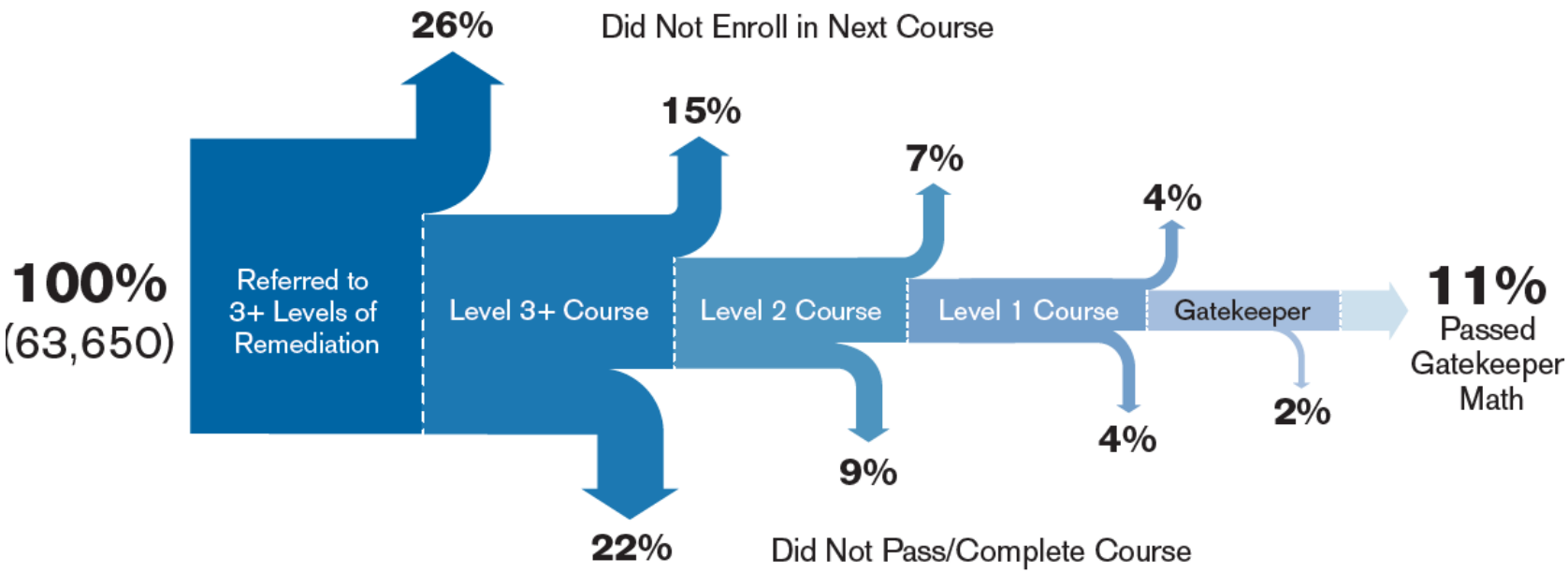
Saxe, K., & Braddy, L. (2015). A Common vision for undergraduate mathematical sciences programs in 2025. *The Mathematical Association of America*. p. 13. Available at http://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf.

A Common Vision (MAA)

- "A primary point emphasized by all the guides is that ***the status quo is unacceptable***."

Saxe, K., & Braddy, L. (2015). A Common vision for undergraduate mathematical sciences programs in 2025. *The Mathematical Association of America*. p. 35. Available at http://www.maa.org/sites/default/files/pdf/CommonVisionFinal.pdf.

Teachers College, Columbia University (2014)



Jaggars, S. S., & Stacey, G. W. (2014). What We Know about Developmental Education Outcomes. Research Overview. *Community College Research Center, Teachers College, Columbia University*. P.5, based on 63,650 students.

Transforming Post-Secondary Education in Mathematics (TPSE)

"Institutions must create multiple pathways into and through the mathematical sciences curriculum, offering a mix of choices for math majors as well as other STEM and non-STEM majors and communicating the wide range of math-related career options."

Transforming Post-Secondary Education in Mathematics: Report of a meeting. *TPSE Math* (2014) p. 13. Available at http://www.tpsemath.org/meeting1

Transforming Post-Secondary Education in Mathematics (TPSE)

"Increase instruction in use of data, statistics, modeling, and computation. Because this is in line with the Common Core Math Standards, it is particularly important in training future teachers."

Transforming Post-Secondary Education in Mathematics: Report of a meeting. *TPSE Math* (2014) p. 13. Available at http://www.tpsemath.org/meeting1

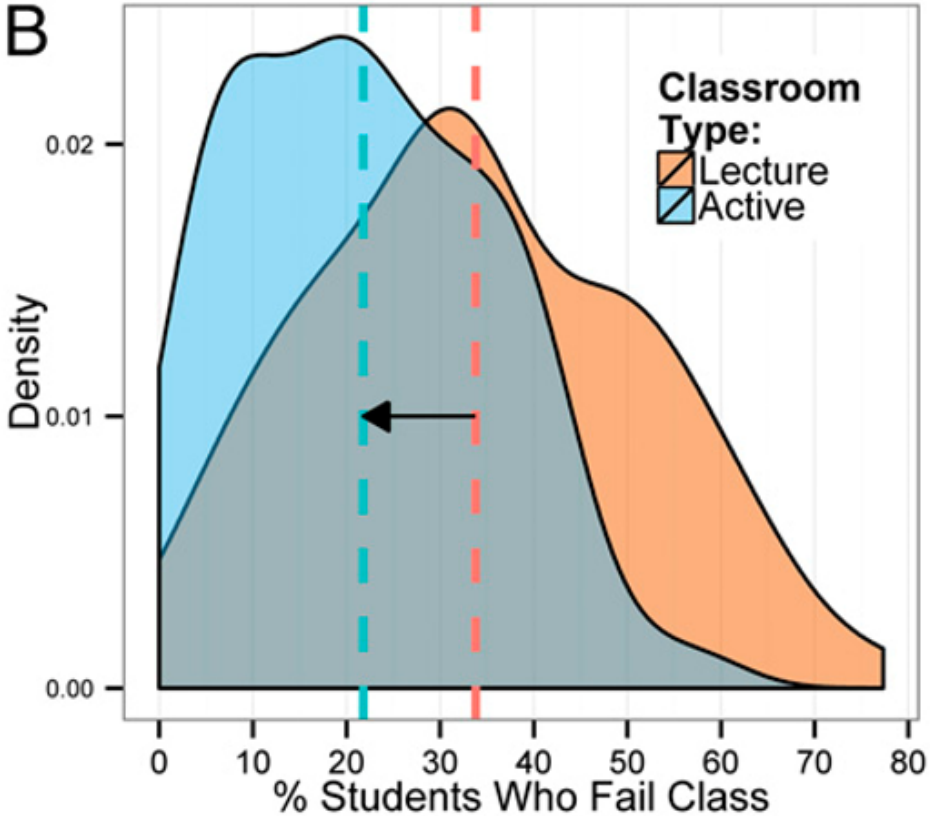
Modernizing Mathematics Pathways at Texas Universities (Dana Center)

"For students majoring in programs such as social or behavioral sciences, the most important mathematics is statistics, not algebra. For liberal arts students, who typically need to take one core math course to graduate in their majors, quantitative reasoning is likely to be more relevant to their future lives and careers."

Kazis, R. & Cullinane, J. (2015). Modernizing Mathematics Pathways at Texas Universities: Insights from the New Mathways Project Transfer Champions. Austin, TX: The Charles A. Dana Center at The University of Texas at Austin. p.4. Available at http://www.utdanacenter.org/wp-content/uploads/modernizing_mathematics_pathways_at_texas_universities.pdf

Dana Center's DCMP

- Principle 1: All students, regardless of college readiness, enter directly into mathematics pathways aligned to their programs of study.
- Principle 2: Students complete their first college-level mathematics requirement in their first year of college.
- Principle 3: Strategies to support students as learners are integrated into courses and are aligned across the institution.
- Principle 4: Instruction incorporates evidence-based curriculum and pedagogy.



Kernel density plots of failure rates under active learning and under lecturing. The mean failure rates under each classroom type (21.8% and 33.8%) are shown by dashed vertical lines. (p.8411)

"lecturing increases failure rates by 55%"

(p. 8412)

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, *111*(23), 8410-8415. http://www.pnas.org/content/111/23/8410.full.pdf

CBMS 2016

"…we call on institutions of higher education, mathematics departments and the mathematics faculty… to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms."

http://www.cbmsweb.org/Statements/Active_Learning_Statement.pdf

Recent shifts:

- Ohio (2015) removed language requiring Intermediate Algebra as the threshold course. They now have Quantitative Reasoning, Statistics and STEM transfer pathways.
- Michigan State (2016) dropped the college algebra requirement.
- California State University (2017) dropped intermediate algebra as a universal prerequisite.

https://www.ohiohighered.org/sites/ohiohighered.org/files/uploads/math/Memo Announcing Endorsed Changes to the OTM Mathematics Guidelines.pdf
https://www.insidehighered.com/news/2016/07/06/michigan-state-drops-college-algebra-requirement
https://edsource.org/2017/cal-state-drops-intermediate-algebra-requirement-allows-other-math-courses/585595

Changing Equations (LearningWorks)

"While the de-emphasis on intermediate algebra remains controversial, the math pathways movement resonates with other initiatives to focus community college education around structured pathways leading toward careers."

Burdman, P. (2013). Changing equations: How community colleges are re-thinking college readiness in math. *Oakland, CA: LearningWorks. p.5. Available at* *http://www.learningworksca.org/wp-content/uploads/2013/10/LWBrief_ChangingEquations_WEB.pdf*

Degrees of Freedom (PACE/ LearningWorks)

- "Access to algebra, in fact, has been considered a civil rights issue. This history leaves many cautious that alternative pathways would create separate but unequal tracks."

Burdman, P. (2015). Degrees of Freedom: Diversifying Math Requirements for College Readiness and Graduation (Report 1 of a 3-Part Series). Policy Analysis for California Education, PACE. p.21. Available at http://edpolicyinca.org/sites/default/files/PACE%20Report%2004222015.pdf

Degrees of Freedom (PACE/ LearningWorks)

- UCLA retired math professor Mark Green: "You have a danger of people being limited throughout their lives by what math they got early on—or didn't. There's a lot of stuff that uses Algebra 2, and students who don't take it may be unaware that they are limiting their options later on."
- "On the other hand," he acknowledges, "it's much better to have someone who genuinely understands modeling and quantitative reasoning and has a feeling for statistics than someone who took an Algebra 2 class but is totally bewildered by it."

Burdman, P. (2015). Degrees of Freedom: Diversifying Math Requirements for College Readiness and Graduation (Report 1 of a 3-Part Series). Policy Analysis for California Education, PACE. p.21. Available at http://edpolicyinca.org/sites/default/files/PACE%20Report%2004222015.pdf

Degrees of Freedom (PACE/ LearningWorks)

"For now, a tentative consensus is emerging among some proponents of alternatives that high school is too early for students to opt out of Algebra 2. Thus, most of the experiments are occurring at the college remedial level."

Burdman, P. (2015). Degrees of Freedom: Diversifying Math Requirements for College Readiness and Graduation (Report 1 of a 3-Part Series). Policy Analysis for California Education, PACE. p.21. Available at http://edpolicyinca.org/sites/default/files/PACE%20Report%2004222015.pdf

Degrees of Freedom (PACE/ LearningWorks)

"Interestingly, both sides of the debate on alternatives are grounded in a concern about eliminating barriers for students: While proponents of alternative pathways view algebra-intensive curricula as a potential barrier to students' success, critics fear that an education without advanced algebra itself constitutes a barrier."

Burdman, P. (2015). Degrees of Freedom: Diversifying Math Requirements for College Readiness and Graduation (Report 1 of a 3-Part Series). Policy Analysis for California Education, PACE. p.22. Available at http://edpolicyinca.org/sites/default/files/PACE%20Report%2004222015.pdf

Reflection

- Pathways

  focused, coherent, rigorous, connection to K12

- Proficiency

  procedural skills, understandings, applications, practices

Calculus I "exams generally require low levels of cognitive demand, seldom contain problems stated in a real-world context, rarely elicit explanation, and do not require students to demonstrate or apply their understanding of the course's central ideas."

Tallman, M. A., Carlson, M. P., Bressoud, D. M., & Pearson, M. (2016). A characterization of calculus I final exams in US colleges and universities. *International Journal of Research in Undergraduate Mathematics Education*, *2*(1), 105-133.

Do your students habitually think:

- Math is a collection of disconnected facts and algorithms
  - **Math is coherent and founded on relationships of ideas.**
- Math is idiosyncratic and situation specific
  - **Math is logical and systematic**
- Math is about quick answer-getting
  - **Math is about authentic problem solving**
- Math is only important for mathematicians
  - **Math is relevant to everyone**
- Math is to be memorized as given
  - **Math is about constructing knowledge**

Heavily Inspired by the taxonomy of the "Views About Mathematics Survey." For more information see: Carlson, M. P. (1999). The mathematical behavior of six successful mathematics graduate students: Influences leading to mathematical success. *Educational Studies in Mathematics*, *40*(3), 237-258. Only one bullet on this list is *not* from Carlson.

Do your students habitually think:

- Math is for the talented few
  - **Math is available to anyone willing to make the effort.**
- My effort has no impact on my ability to learn math: I am not a math person.
  - **My achievement depends on my persistence.**

Heavily Inspired by the taxonomy of the "Views About Mathematics Survey." For more information see: Carlson, M. P. (1999). The mathematical behavior of six successful mathematics graduate students: Influences leading to mathematical success. *Educational Studies in Mathematics*, *40*(3), 237-258. Only one bullet on this list is *not* from Carlson.

Thank you!

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